Question Paper Code : 6622

BCA (Semester-I) Examination, 2021

MATHEMATICS - I

[Paper : BCA-103]

Time : Three Hours] **[Maximum Marks : 70**

Note : Answer **any five** questions. Each question carries **equal** marks.

1. (a) Find by elementary row transformation the inverse of the matrix : [7]

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

(b) Find the rank of the matrix : [7]

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

2. (a) Discuss the consistency of the following system of equations : [7]

(b) Using Stoke's theorem or otherwise, evaluate : [7]

$$\int_c \left[(2x-y)dx - yz^2dy - y^2zdz\right]$$

where c is the circle $x^2+y^2=1$, corresponding to the surface of sphere of unit radius.

9. Find the eigenvalues and the corresponding eigenvectors of the matrix : [14]

$$A = \begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$$

10. (a) Using Beta and Gamma function, evaluate [7]

$$\int_0^1 \left(\frac{x^3}{1-x^3}\right)^{\frac{1}{2}} dx$$

(b) Use Divergence theorem to evaluate $\iint_S \vec{F}\cdot d\vec{S}$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface bounding the region $x^2+y^2=4$, $z=0$ and $z=3$. [7]

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$$2x + 3y + 4z = 11$$

$$x + 5y + 7z = 15$$

$$3x + 11y + 13z = 25$$

If found consistent, solve it.

(b) Reduce the matrix 'A' to its normal form, when : [7]

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

3. (a) Find the nth differential coefficient of [7]

$$y = \frac{1}{a^2 - x^2}$$

(b) If $u = e^{xyz}$, find the value of $\frac{\partial^3 u}{\partial x \partial y \partial z}$. [7]

4. (a) If $y = \log\left(x + \sqrt{1 + x^2}\right)$, prove that :

$$\left(1 + x^2\right) y_{n+2} + (2n+1) x y_{n+1} + n^2 y_n = 0.$$ [7]

(b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, prove that : [7]

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

5. Obtain Taylor's expansion of $\tan^{-1}\frac{y}{x}$ about $(1, 1)$ upto and including the second degree terms. Hence compute $f(1.1, 0.9)$. [14]

6. If u, v, w are the roots of the equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ, find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. [14]

7. (a) Find the value of 'n' for which the vector $r^n \vec{r}$ is solenoidal, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. [7]

(b) Find the constant a, b, c so that $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational. [7]

8. (a) Using Green's theorem, evaluate $\int_c \left(x^2 y\,dx + x^2 dy\right)$, where c is the boundary described counter clockwise of the triangle with vertices $(0, 0), (1, 0), (1, 1)$. [7]